X-bar Structure & the Golden String

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- The study of complex systems seems to affirm the Thompson-Turing claim that “some physical processes are of very general occurrence.”
- Notably, those involving Fibonacci-based “golden” forms, ubiquitous in nature.
- This lends immediate interest to the observation that the repeated structural motif in the human syntactic system, the X-bar schema, is likewise a “golden” form (Medeiros 2008),
- and leads us to inquire whether whatever is behind the natural ubiquity of such phenomena, in other domains, might possibly be at work here as well.
- If so, this peculiar aspect of human phrase structure would fall under Chomsky’s (2005) “third factor”, a fact about language which is neither encoded in the particulars of our genome, nor learned from the environment, but determined by domain-general principles beyond the organism.

“Golden” Mathematics

- The Golden number (aka the golden ratio, the golden section, the golden mean)
- is *x*, such that $x^2 - x - 1 = 0$ $x/(x+1) = 1/x$
- ~ 1.618… Phi
- (or sometimes its reciprocal, ~0.618… phi)
- The Golden angle, associated with the dominant spiral mode of phyllotaxis, is just phi measured out on a circle.
- It is the *most irrational* number.
- Intimately linked with the Fibonacci sequence and the Golden String (~Fibonacci word).

Fib #: $a_n = a_{n-1} + a_{n-2}$ *1,1,2,3,5,8,13…*
Fib word: $s_n = s_{n-1}\ s_{n-2}$ *1011010110110…*

- The Fibonacci numbers, and the closely linked Fibonacci word (aka Golden String) in particular, are important topics in the study of symbolic dynamics, physics, theoretical computer science, etc.
- These patterns have a number of 'special' mathematical properties.

Fib #'s, addition:

1,1,2,3,5,8,13,…

- 1+1=2
- 1+2=3
- 2+3=5

Fib words, concatenation:

(0),1,10,101, 10110, 10110101,

0, 1: 10

1, 10: 101

101, 10: 10110

Fibonacci in brain & cognition

- **Perception**: Tom Bever at the University of Arizona has experimental results indicating visual frames with sides in the golden ratio maximally facilitate perception of depth and motion.
- **Optimal sub-cellular computation**: Koruga (1986) observes that eukaryotic cytoskeletal microtubules are composed of dimers in a cylinder with Fibonacci (5, 8) parastichy arrangement, and demonstrates that this provides optimal 'transmission fidelity' if the system is conceived of as a cellular automaton passing information (the different states of the cells being the two possible states of the dimers).
- **Frequency packing**: Roopun et al (2008) observe that EEG frequencies cluster, not according to a natural log distribution, but one based on the golden ratio. As that ratio is the *most irrational number*, this frequency distribution minimizes interference between different signals:
  - "[…] in using phi as a common ratio between adjacent frequencies in the EEG spectrum (Figure 1), the neocortex appears to have found a way to pack as many, minimally interfering frequency bands as possible into the available frequency space."

Fib in language

- Uriagereka (1998) claims that the cross-linguistic **possibility and frequency of syllable types** falls out from his “F-game”, a concatenative Fibonacci pattern.
- Idsardi & Uriagereka (2009) observe that metrical **possibilities for footing** also follow a Fibonacci sequence.
- Carnie & Medeiros (2005), Medeiros (2008) observe the Fibonacci numbers in **X-bar phrase structure.**

The Golden Phrase

- I suggest that we should add to the family of related "golden" mathematical objects (the golden number/section/mean; the golden angle, the golden string)
- The "Golden Phrase", i.e. the X-bar schema.
- In essence, this phrasal shape is the expression, in binary-branching syntactic trees, of the very same Fibonacci theme.
- This kind of phrasal organization has a number of "special" properties…

The Golden Phrase is special

- In what follows, I will point to three considerations which pick out this kind of phrase structure as special:
  - (1) The X-bar schema is the simplest kind of syntactic (multi-)fractal.
  - (2) The X-bar schema is the minimal semantic generator, the first shape to unlock the full set of predicate-argument meanings.
  - (3) X-bar grammar, with specifier-head-complement order, yields strings related to the Golden String (infinite Fibonacci word); it has the lowest ambiguity among "binary generators of binary".

The X-bar schema.

Of all the ways that syntactic structure could be built up, one particular ‘growth solution’ seems to dominate in natural language.

This is the so-called *X-bar schema*:

XP = [ZP [$\mathbf{X^0}$ YP]]

In words: a phrase of any type (a verb phrase, noun phrase, whatever, thus an XP(hrase)) is built around a head (X^0), with asymmetrically arranged ‘slots’ for two additional phrases of the same shape.

NP = [The barbarians’ [**destruction** [of Rome]]

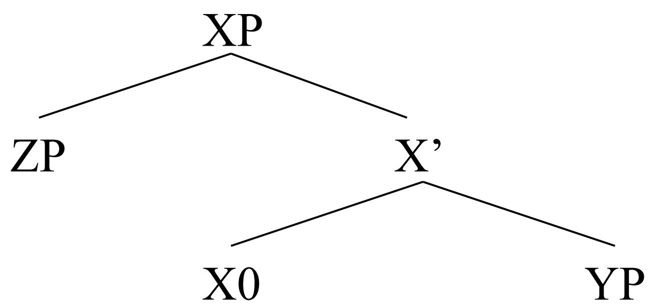
Sentence = [The barbarians [**destroyed** [Rome]].

The phrasal off-branches (YP, ZP) may be expanded indefinitely:

[The ravening hordes of barbarians] [**destroyed** [the gleaming city on the hill]], and so on.

Background: [Spec [Hd Comp]]

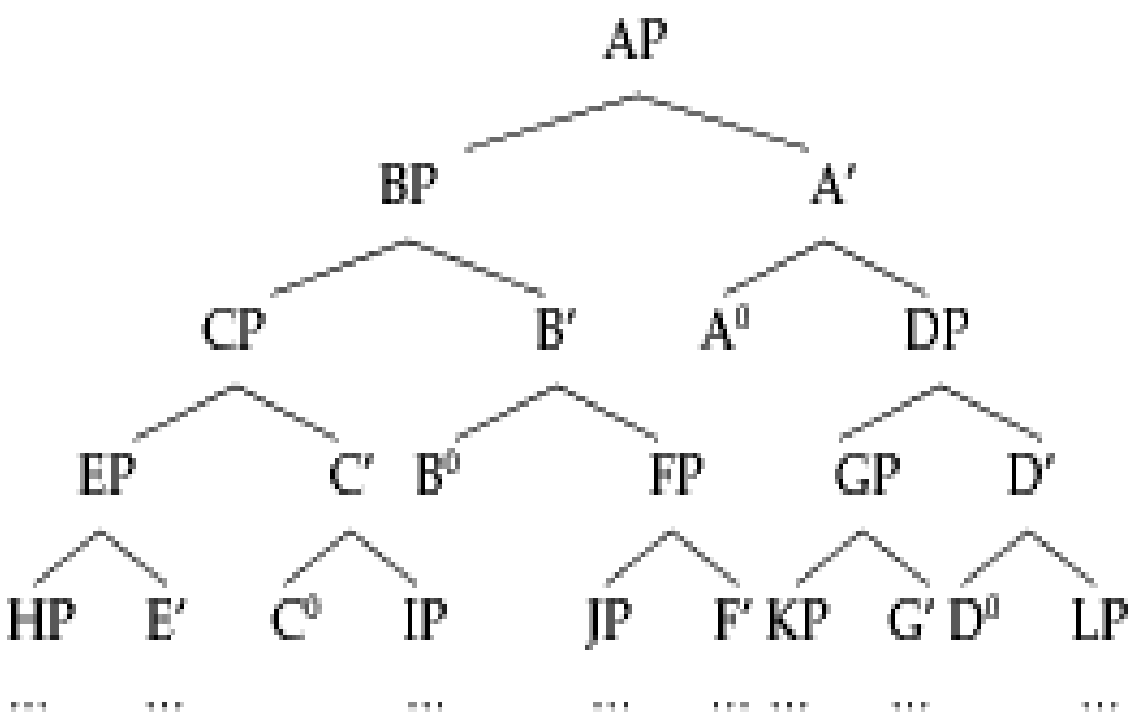
- Since Chomsky (1970), it is widely held that the syntactic structures of natural language are constructed around the ‘X-bar molecule’ shown below:



- The claim is that one finds phrases (XPs) of only the following shapes:
  a. XP = X^0
  b. XP = [X^0 YP]
  c. XP = [ZP [X^0 YP]]
- One does not find ‘exocentric’ phrases such as
  *XP = [YP ZP] (*contra* Starke 2004)
- Nor phrases with more than a single complement and specifier:
  *XP = [WP [ZP [X^0 YP]]] (*contra* Chomsky 1995a)
- Nor phrases in which the head (X^0) is not at the ‘bottom’:
  *XP = [X^0 [YP ZP]] (but see Moro 2000, Pereltsvaig 2006, below)

Fibonacci in X-bar

(1) In the maximally-expanded X-bar tree, the number of XPs, X’s, and X0s at successive depths each follows the Fib sequence.



XP	*X′*	*X⁰*
1	*0*	*0*
1	*1*	*0*
2	*1*	*1*
3	*2*	*1*
5	*3*	*2*
… *Fib*(n)	*Fib*(n–*1*)	*Fib*(n–*2*)

Alternative phrasal arrangements…

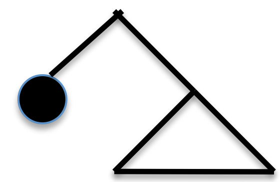
- What follows is concerned with showing that the X-bar schema has a lot to recommend it, when compared against other ways that phrases might be assembled.
- We'll therefore need to consider what else is possible -- what else could phrases look like?
  – Take phrases to be recursive 'recipes' for structure building.
  – Assume binary branching (Kayne 1984).
  – Phrases enable discrete infinity; concretely, they contain **terminals**, and **non-terminals**.

Simplest 'molecule' of structure: one level of embedding.

If we restrict possibilities to a single layer of syntactic combination, only one shape yields discrete infinity:

- The Spine, Phrase = [terminal Phrase].

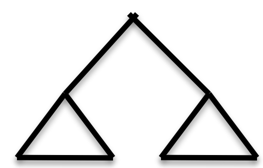
  1 → 0 1



The other naïve possibilities,

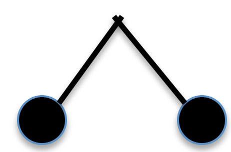
- Phrase = [Phrase Phrase]

  1 → 1 1



- or Phrase = [terminal terminal],

  1 → 1 0

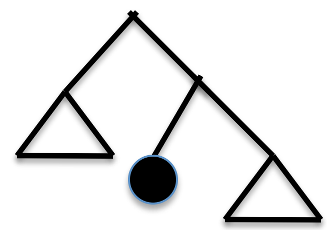


obviously could not serve as bases for a language-like system.

Next class

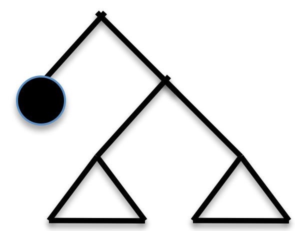
- Adding another layer of embedding within the repeated molecule opens up three new possibilities (plus variations/ rotations):
- **X-bar**, 2 → 2 1, 1 → 0 2,

Phrase = [Phrase [terminal Phrase]]



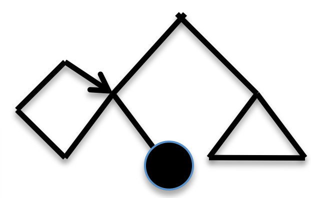
- **D-bar**, 2 → 0 1, 1 → 2 2,

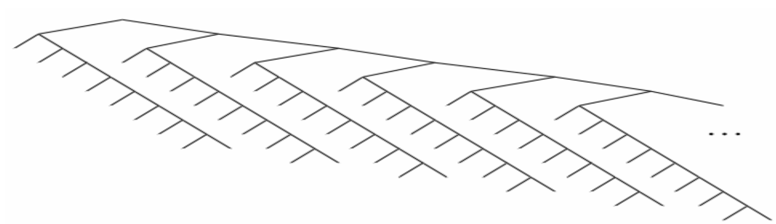
Phrase = [terminal [Phrase Phrase]]

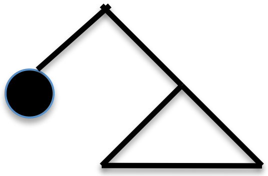
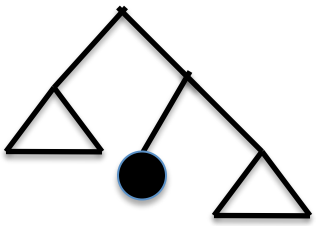
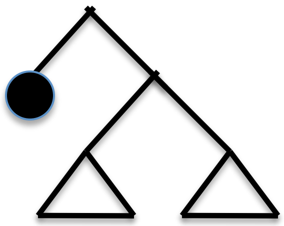
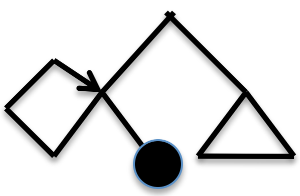
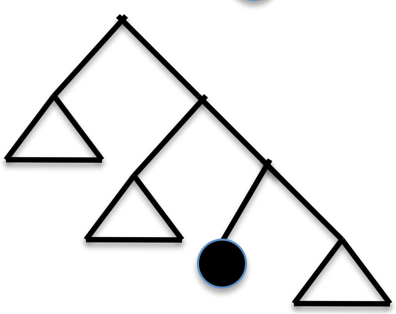


- **Spine of Spines**, 2 → 1 2, 1 → 0 1

Phrase = [[terminal sub-Phrase] Phrase]





System	*Tree*	*Matrix*	*Recurrence relation*	*Growth Factor*
Spine		$\begin{bmatrix}1\end{bmatrix}$	$a_n = a_{n-1}$	1
X-bar		$\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$	$a_n = a_{n-1} + a_{n-2}$	Phi ~ 1.618
D-bar		$\begin{bmatrix}0 & 1\\2 & 0\end{bmatrix}$	$a_n = 2a_{n-2}$	√2 ~ 1.414
Spine of Spines		$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$	? (complicated)	1
3-bar		$\begin{bmatrix}1 & 1 & 0\\1 & 0 & 1\\1 & 0 & 0\end{bmatrix}$	$a_n = a_{n-1}+a_{n-2}+a_{n-3}$	~1.839 “Tribonacci constant”

Next: Fractals & the Cantor Set

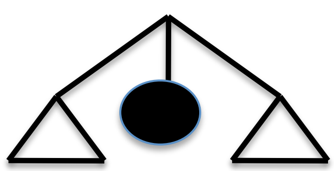
Fractals are self-similar objects of non-whole-number dimension; their "size" depends on the scale at which they are measured.

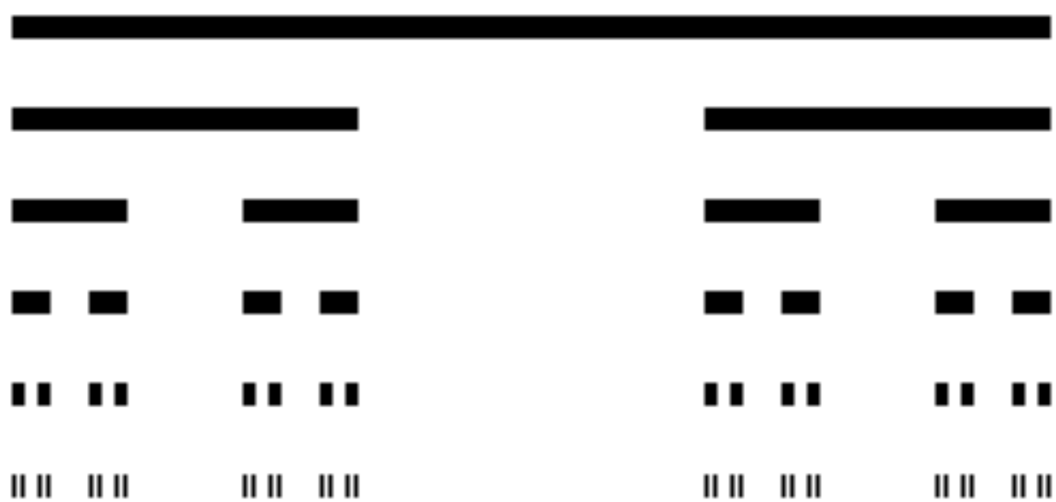
The Cantor set is formed from a line segment, by removing the middle third, then middle thirds of the remainders...

This is **the *simplest* fractal**:

- Background dimension cannot be lower than a 1-dimensional line.
- Division in thirds is the first scheme yielding a fractal.

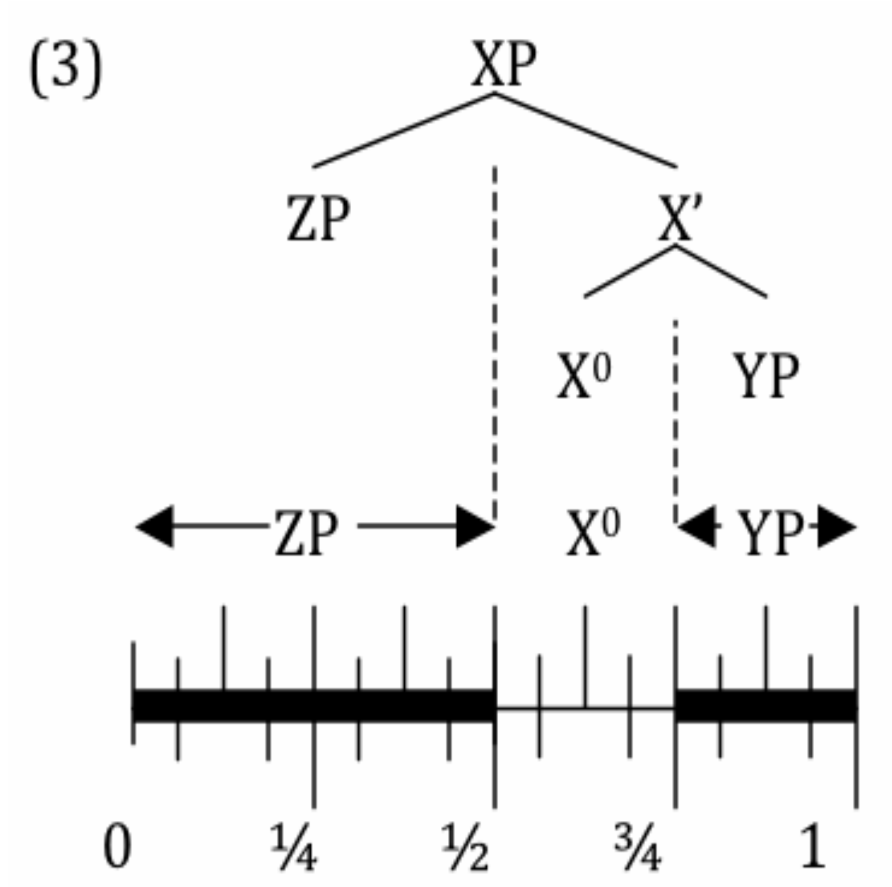
The self-similarity here invites a kind of *phrasal* analysis: within each "generation", there are two copies of the whole, and one "dead end" (deleted segment ~terminal):





X-bar schema as (multi)fractal

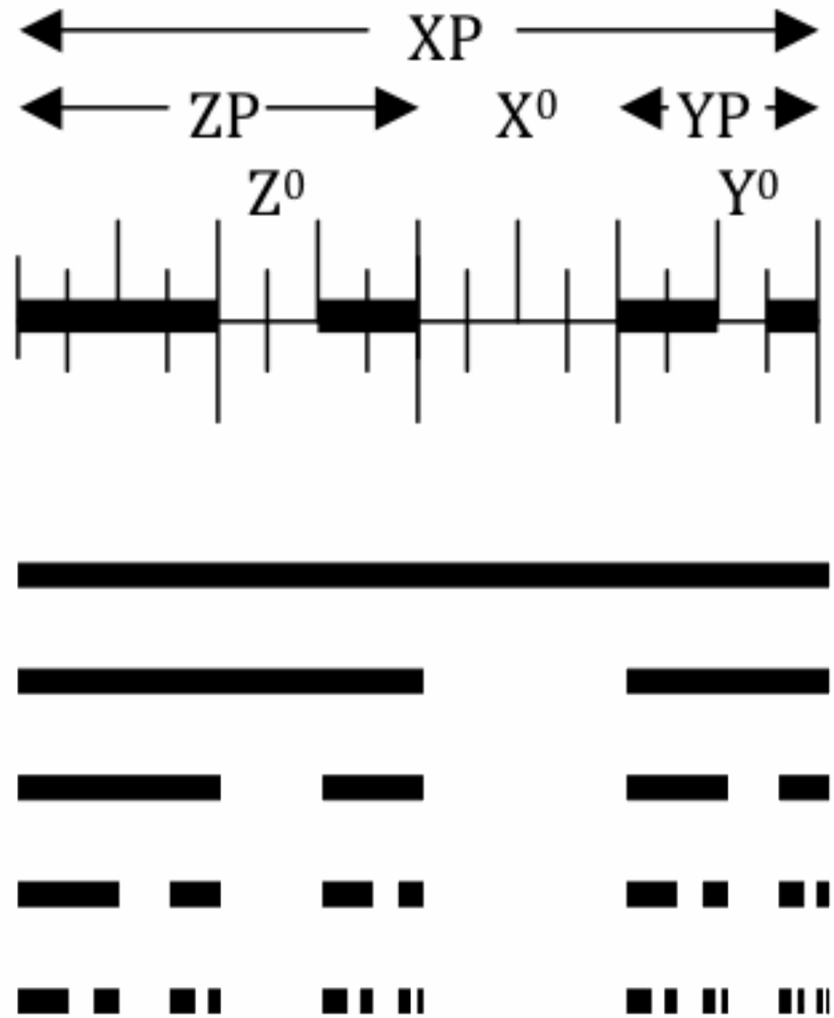
- Consider mapping the X-bar schema to a line segment,
- Such that binary branching in the syntactic form corresponds to geometric halving,
- And heads/terminals corresponding to deleting a line segment.



And so on: fractal structure

- Of course, ZP and YP themselves have the same internal structure as XP:

- Continued indefinitely, this produces an asymmetric (or two scale) Cantor set.
- Each generation has one ½ and one ¼ scale copy of the whole.



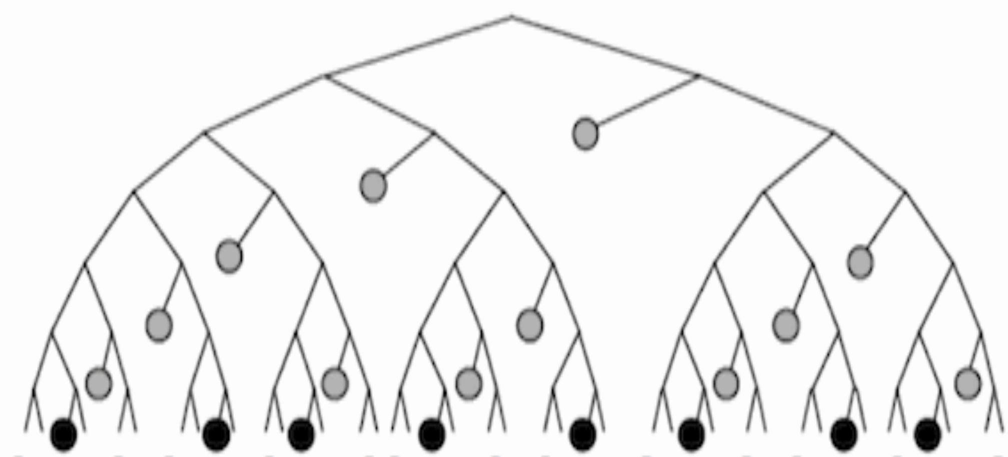
Asymmetric Cantor set ~ X-bar tiling

As a fractal, this has a number of properties worth mentioning.

It is the simplest kind of syntactic fractal; i.e. it is the smallest kind of self-similar binary-branching object whose non-terminal image on the line is neither the full line, nor a single point.

Its (Hausdorff) dimension is $\log_2(\text{Phi})$ ~.694

It is actually a multi-fractal (each unit of structure contains two copies of the whole, at different scales (½ & ¼).



X-bar schema and semantic expressive power

- There is another reason to think that the X-bar form is “special”
- Related to its expressive power when mapped to semantic interpretation.
- The X-bar schema is the *minimal semantic generator.*
- In the sense that predications of any internal structure, stated with predicates of arbitrary adicity,
- Can be expressed in a X-bar syntactic form (utilizing the syntactic equivalent of Schoenfinkelization/Currying).
- But no simpler form will do.
- In other words, the X-bar schema is *just right*: just big enough to get the job done (i.e. to express any kind of predication) -- having a larger phrasal shape doesn’t buy you any additional expressive power.

Bicomplex predication

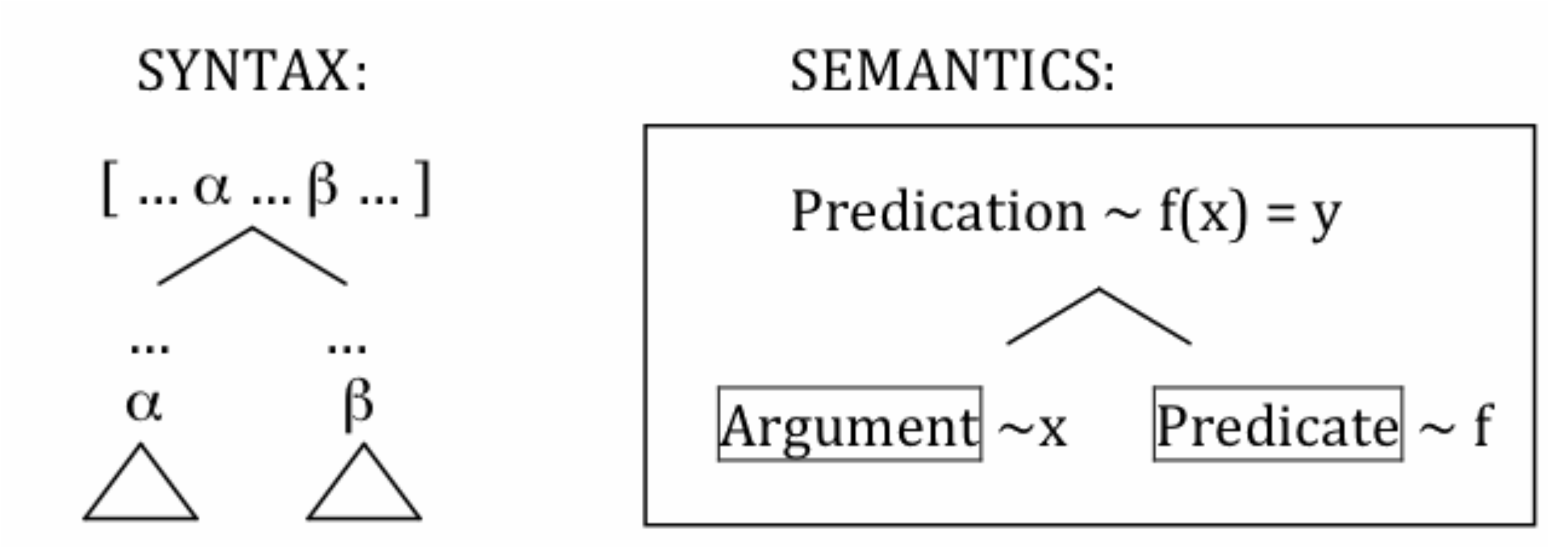
- Note the following about predicate-argument structure in natural language:
- Both predicates and arguments may contain further predicate-argument structure:

Simple: $[_{Arg}$ The people] $[_{Pred}$ know].

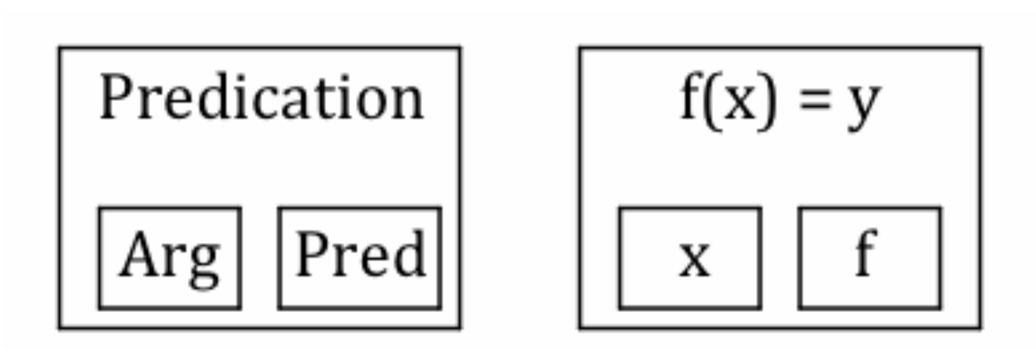
Complex Arg: $[_{Arg}$ The people $[_{Arg}$ you] $[_{Pred}$ met t]]
$[_{Pred}$ know].

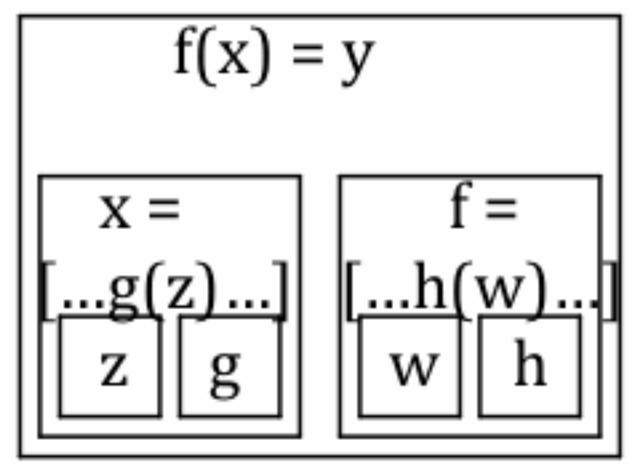
Complex Pred: $[_{Arg}$ The people]
$[_{Pred}$ know $[_{Arg}$ you] $[_{Pred}$ exist t]].

Bi-Complex: $[_{Arg}$ The people $[_{Arg}$ you] $[_{Pred}$ met t]]
$[_{Pred}$ know $[_{Arg}$ you] $[_{Pred}$ exist t]].



- I adopt the minimal notion of compositionality depicted above.
- Bicomplexity in semantic composition requires bicomplex syntax.
- Bicomplex syntax: two(+) growth points per molecule.
- So, at least as complex as X-bar (or D-bar?).

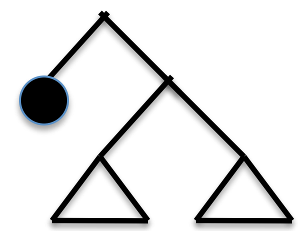




(A)Symmetry, Fractal Semantics?

- Conceivably, the structural asymmetry inherent in the dominant X-bar form is “recruited” for the semantic asymmetry between predicates and arguments.
- Making it more *useful*, in a sense: not only do you get bicomplex semantics, you get **asymmetric bicomplexity**, a basis for Fregean semantic asymmetry.
- A crucial case here is the structure of the copula, argued to be this (Moro 2000, Pereltsvaig 2006):

[cop [$_{SC}$ XP YP]]

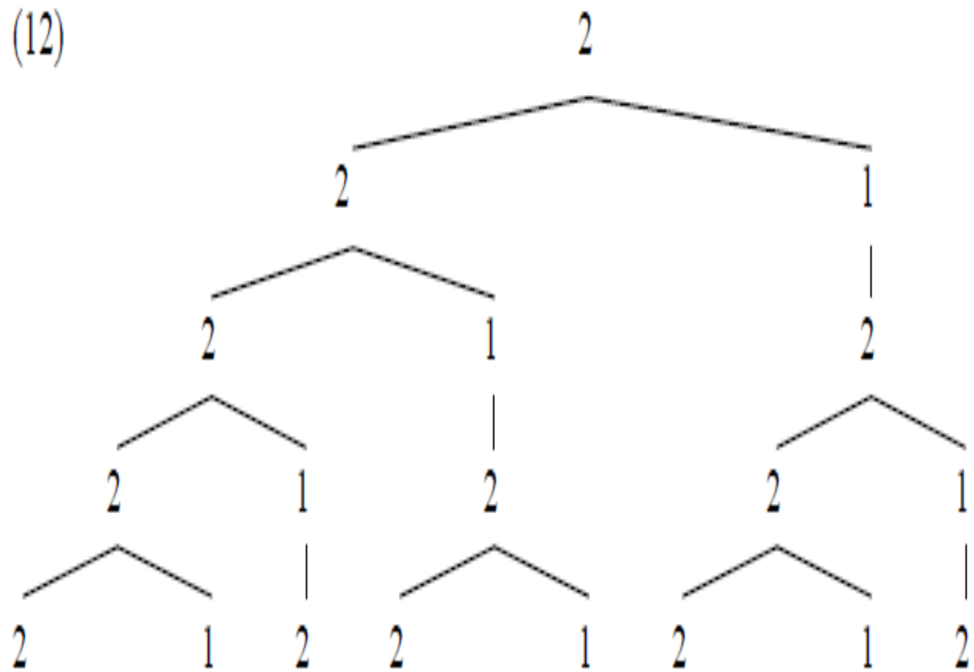


- This is a (partial) manifestation of X-bar’s obscure sibling, the apparently rare D-bar configuration.
- Crucial property: symmetry between the two objects combined (the two growth points in the phrasal shape).
- Here, we have a symmetric syntactic form just where we need it to construct a symmetric meaning (equation).

The Golden String and X-bar

- As is well known, the geometrical structure of a strictly binary-branching tree can be described with no loss of information by pruning all the terminals and taking just the (mixed unary- and binary-branching) non-terminal tree.
- Consider what happens when we do this ‘compression’ to the maximal X-bar tree…

Rabbit tree



- This is just the familiar 'rabbit tree', the problem Fibonacci was thinking about.

X-bar: 2 = XP, 1 = X'

Rabbits: 2 = adult pair, 1=juvenile pair

- As this tree is grown as an L-system, successively longer portions of the same sequence (the Fibonacci word) are generated.

Self-generating procedure for the Golden String

{examine the value at a pointer.

If val=1, append 10 to the end of the string.

If val=0, append 1 to the end of the string.

{ Move the pointer one space right.

Repeat.

- Begin with just the first two digits of the GS (10), with pointer on the second digit (0, underlined and bolded):

1 **0**

- The pointer is at 0, so we add 1 to the end and move the pointer.

1 0 **1**

- Now the pointer is on 1; we add 10 and move the pointer.

1 0 1 **1** 0

- And so on:

1011**0**10, 10110**1**01, 101101**0**110, 10110101**1**101, 1011010110110, etc.

The Golden string encodes its own computation

- The Golden String has a fascinating property of ‘vertical’ self-similarity at many scales.
- The sequence encodes the very procedure used to compute the sequence…
- Idea: perhaps this is significant in light of the **double articulation** of language noted since antiquity: its dual life as a linear outer form and a hierarchically structured inner form (strings and trees, basically).
- This object, in a sense, *brings its own double articulation with it*; the projection of a syntactic form from its sequence is inherently already there.
- In other word: there’s already a tree in this string.

- http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibrab.qt

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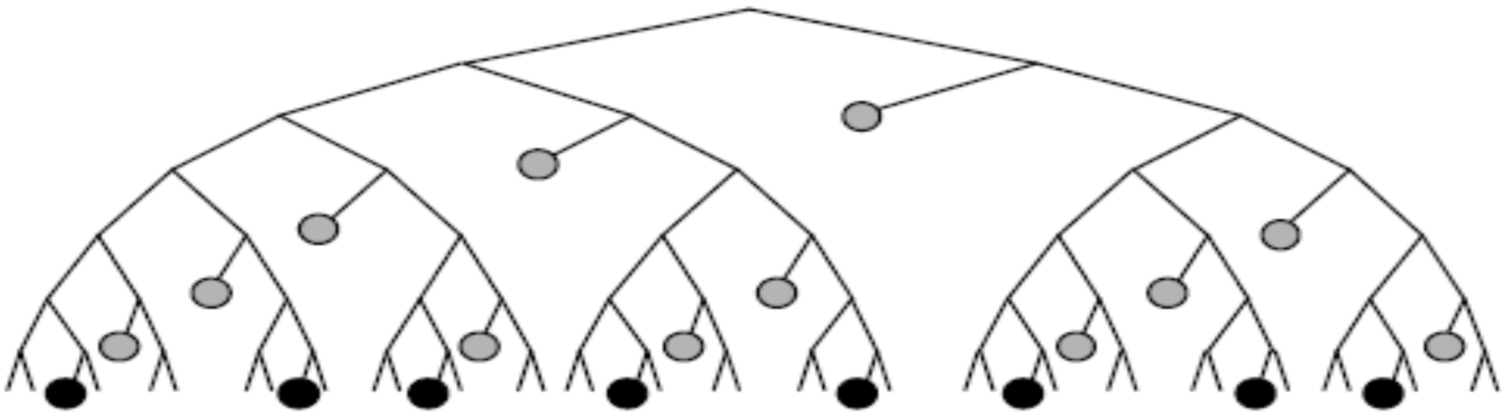


(1) l s l l s l s l l s l l s l s l l s l s l

(2) S c c S c S c c S c c S c S c c S c S c c

(1) lsllslsllsllslsllslsl

(2) -SccScSccSccScSccScScc

(3) 101101011011010110101

(1) lsllslsllsllslsllslsl

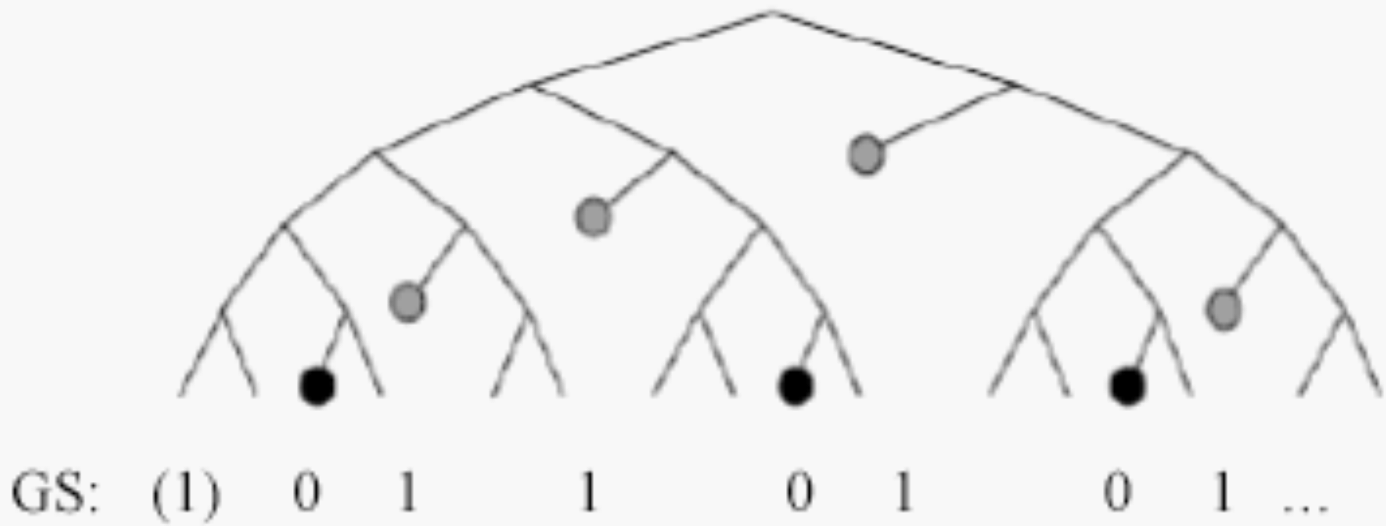
(2) -SccScSccSccScSccScScc

(3) 101101011011010110101

- So: the sequences of large/small open categories on the bottom ‘frontier’ of the partial expansion of the maximal X-bar tree follow the golden sequence exactly (1).
- …in fact, are successive Fibonacci length portions of that object.
- The classification of those open categories as specifiers or complements is a slightly different pattern (2).
- The spec/comp sequence also follows the golden string pattern, but starts at index 2.
- Likewise, if one examines the sequence of head positions, marked for whether they are introduced in the last generation (bottom line of the tree) or not, one also finds the GS starting at index 2.

In the abstract, ‘open’ structure, this sequence emerges in the relative depth of head positions, significant for prosodic prominence.

24) Golden String in ‘bottom-ness’ of X-bar heads: deepest=0, higher=1



This is reflected in surface prosodic contour:
deepest elements are local stress peaks (Chomsky & Halle 1968, Bresnan 1971, 1972 Cinque 1993).

The Golden Parse?

- One interesting property of the golden string is that, although it is generated by a total rewrite (1→ 1 0, 0→ 1) in each expansion…
- The sequence is always the same, no matter how far it has been expanded.
- ***Plus ça change, Plus ç'est la même chose…***
- This may be significant for parsing.
- Since the sequence is always the same from the front end, one can (i) detect deviations, 'pushing' or 'popping' to another depth of structure, and (ii) assign structure to a string *as it comes in*, without waiting to the end.

A sentence sounds like *this*…

- Thus, if the tree is of uniform depth, one *knows how the tune should go*, so to speak.
- If the tree is not of uniform depth—the typical case—then deviations from the 'expected' sequence in effect provide a roadmap of changes in tree depth,
- and do not overly disrupt the mapping between string and hierarchy (due to the pervasively self-similar nature of the Golden String).

Why SHC?

- Mirror order linearization (comp-head-specifier, CHS) of X-bar structures would yield strings which are *backwards* portions of the Golden String; thus SHC X-bar and CHS X-bar have equivalent structural ambiguity in a static sense, over complete strings.
- But SHC X-bar is more useful for *dynamic* recovery of structure from strings.
- It is easier to compare an incoming sequence, bit-by-bit against a known standard, from an invariant beginning up to a variable ending (as for SHC X-bar), rather than from variable ending backwards to invariant beginning (as for mirror-order CHS X-bar).
- On the other hand, ‘mixed’ X-bar linearizations (SCH, HCS) would fail to form the relevant golden sequence.

Conditions for a "language-like" phrase structure system.
Binary alphabet; 1 → *x y*, 0 → *z w*; *x,y,z,w* in {0, 1, *}

A: Termination. At least one of x, y, z, w is terminal (/null). Thinking of these systems as a (highly abstract!) basis for something like language, we want them to be discrete, built around lexical atoms. Thus, at least one branch in the system must introduce a terminal.

B. Completeness. At least one of x..w is 1, and at least one is 0. In other words, each non-terminal occurs an infinite number of times as the tree is infinitely expanded.

C. Mixed Loop. 1 → ...0... and/or 0 → 1. Along the same lines as the above condition, we want to ensure that there are not two disjoint loops, with 1s only occurring under other 1s, 0s only occurring as descendants of 0s. If that were true, since there will only be one start symbol, one or the other loop will never be introduced, hence is spurious.

D.? Bi-complexity. Not both 1 → ...*... and 0 → ...*... We probably also want to rule out things that are unary-branching; it seems they can have absolute recoverability (at least if they are not just unary, but unidirectionally, branching), but they clearly lack the 'semantic power' of something like X-bar. For instance, they allow no mapping of constituents to predicate/argument structure such that both predicates and arguments can each contain further predicate/argument pairs. For the same reason, we will rule out systems including a rule 0/1 → * *, since that will produce a 'double headed spine'.

The table below is all conceivable binary phrase structure systems; those ruled out as “unreasonable” are color-coded for the condition they violate.

What’s so special about this one, the Fibonacci grammar associated with Spec-Head-Comp X-bar?

0: 0 0 1: 0 0	0: 0 1 1: 0 0	0: 0 * 1: 0 0	0: 1 0 1: 0 0	0: 1 1 1: 0 0	0: 1 * 1: 0 0	0: * 0 1: 0 0	0: * 1 1: 0 0	0: * * 1: 0 0
0: 0 0 1: 0 1	0: 0 1 1: 0 1	0: 0 * 1: 0 1	0: 1 0 1: 0 1	0: 1 1 1: 0 1	0: 1 * 1: 0 1	0: * 0 1: 0 1	0: * 1 1: 0 1	0: * * 1: 0 1
0: 0 0 1: 0 *	0: 0 1 1: 0 *	0: 0 * 1: 0 *	0: 1 0 1: 0 *	0: 1 1 1: 0 *	0: 1 * 1: 0 *	0: * 0 1: 0 *	0: * 1 1: 0 *	0: * * 1: 0 *
0: 0 0 1: 1 0	0: 0 1 1: 1 0	0: 0 * 1: 1 0	0: 1 0 1: 1 0	0: 1 1 1: 1 0	0: 1 * 1: 1 0	0: * 0 1: 1 0	0: * 1 1: 1 0	0: * * 1: 1 0
0: 0 0 1: 1 1	0: 0 1 1: 1 1	0: 0 * 1: 1 1	0: 1 0 1: 1 1	0: 1 1 1: 1 1	0: 1 * 1: 1 1	0: * 0 1: 1 1	0: * 1 1: 1 1	0: * * 1: 1 1
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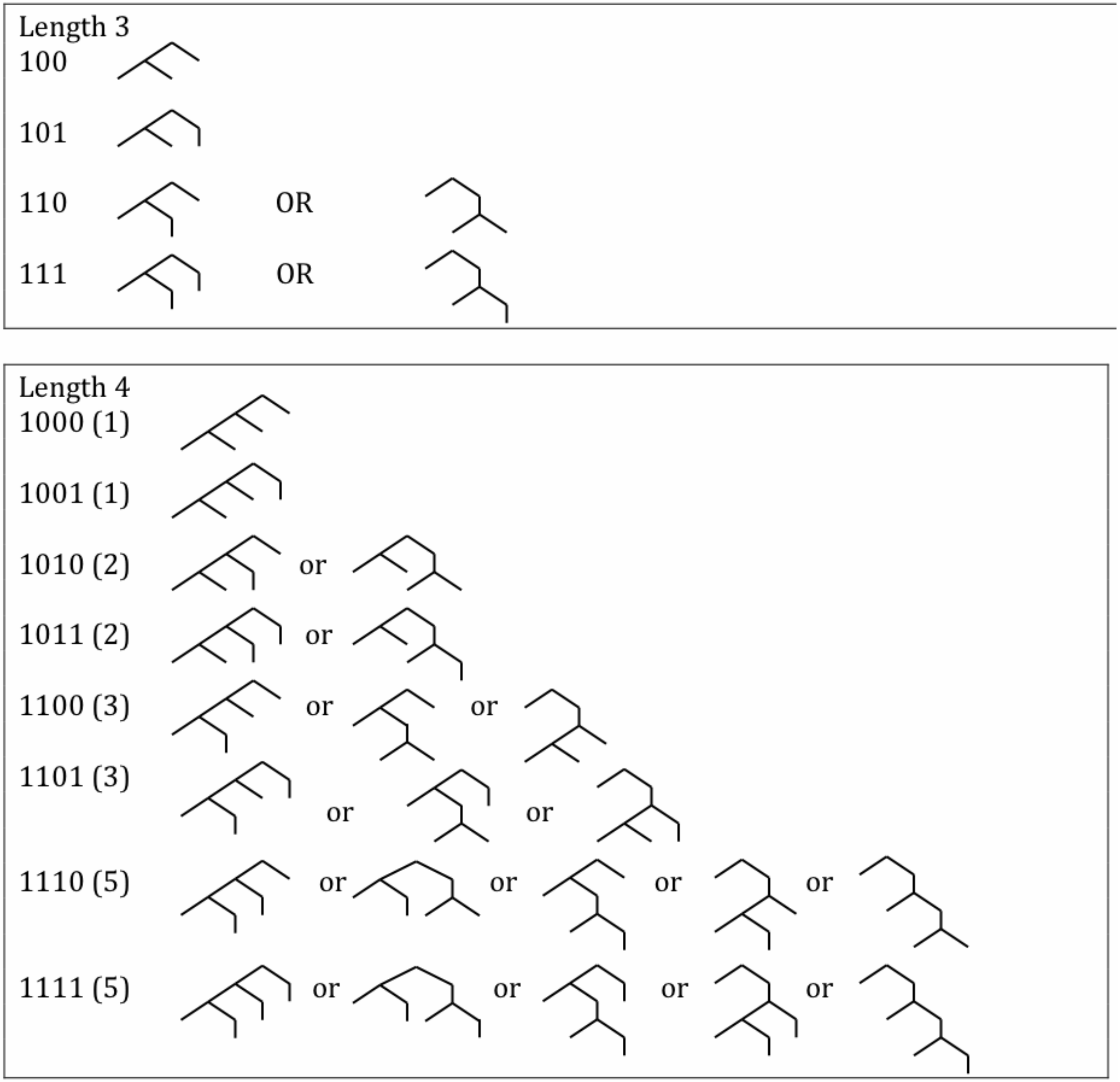
0: 0 0 1: * 0	0: 0 1 1: * 0	0: 0 * 1: * 0	0: 1 0 1: * 0	0: 1 1 1: * 0	0: 1 * 1: * 0	0: * 0 1: * 0	0: * 1 1: * 0	0: * * 1: * 0
0: 0 0 1: * 1	0: 0 1 1: * 1	0: 0 * 1: * 1	0: 1 0 1: * 1	0: 1 1 1: * 1	0: 1 * 1: * 1	0: * 0 1: * 1	0: * 1 1: * 1	0: * * 1: * 1
0: 0 0 1: * *	0: 0 1 1: * *	0: 0 * 1: * *	0: 1 0 1: * *	0: 1 1 1: * *	0: 1 * 1: * *	0: * 0 1: * *	0: * 1 1: * *	0: * * 1: * *

Static ambiguity of binary generators of binary understood as term-rewriting systems

- In terms of ambiguity of complete output strings (with * null), X-bar as understood above is one of 16 possibilities of its ‘size’ (the others are alternative linearizations of X-bar and the other ‘reasonable’ 3-type systems).
- In that group, the possibilities fall into three equivalence classes:
- The class containing the GS/X-bar form has the lowest ambiguity for the cases I’ve worked out: for a string length n, there are two unambiguous strings, two maximally ambiguous (full Catalan number of analyses) strings, and some number of intermediately-ambiguous strings.
- Another class (D-bar) accepts every string of a given length, and assigns the full Catalan number of analyses to each.
- The third class (Spine of Spines) accepts only a single string of each length, but assigns an infinite number of possible analyses to it.



Left: strings accepted by Fibonacci grammar 1→ 10, 0 → 1 (~X-bar), and analyses assigned to each; lengths 3 and 4.

D-bar as term-rewriting system: full Catalan ambiguity.

1 → 0 0

0 → 1

This grammatical system accepts any binary string, and assigns the full Cataln number of possible analyses to each.

Length 3

000

001

010

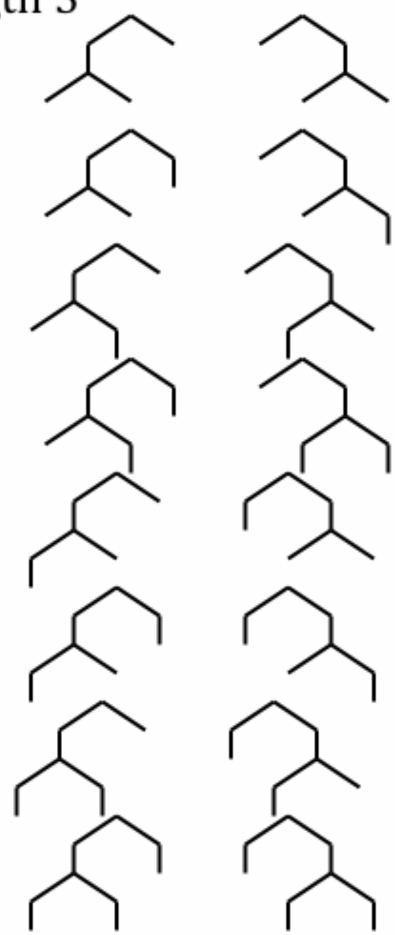
011

100

101

110

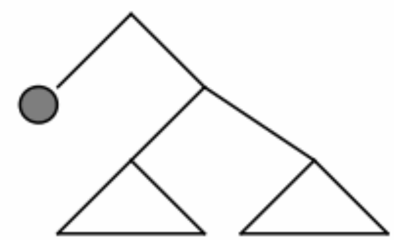
111



2 → 0 1

1 → 2 2

$$\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$$



Reasons for caution

- Comparing “online” ambiguity in these systems is still work in progress.
- Not so stunning: it belongs to the best of three equivalence classes, quite a limited field.
- And X-bar only wins out over competitors if we conceive of these as total-rewrite systems.

- The intuition explored here seems to clash with results from computer science.
- The Golden String is well-studied in that field. It is considered a **worst-case for application of efficient pattern recognition algorithms** (Aho 1990).
- That sounds like the *opposite* of what I'm on about here.
- But I think the results can be reconciled, in light of the observation that pattern-recognizers have variable performance on different input strings in essence because they exploit regularities in strings to "skip over" things that can safely be ignored.
- The Golden String/Fibonacci word isn't chaotic, but neither is it predictably regular; it is *quasi*-periodic.
- Moreover, despite the general interest of the problem from other applications, I'm not sure that *finding all occurrences of a given word in a string* is relevant to the problem under consideration here.

Towards explaining the Golden Phrase: *Why is language that way?*

- I would like to suggest that the following themes are involved in picking out the X-bar form:
  - Its unique (multi)fractal properties
  - Asymmetric bicomplexity
  - Self-similarity, quasi-periodicity

From the present perspective, investigation of the syntax of apparent departures from the X-bar structural format (esp. the copula; maybe coordination) is of top importance.

(Fanciful?) idea: the shunned competitor (D-bar) is something like Lucas patterns in phyllotaxis, possible but disfavored by the dynamics of the system.